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LQR-PID Control Applied to Hexacopter Flight

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Abstract: In this paper the mathematical model representing the dynamic of a Unmanned Aerial Vehicle (UAV) is studied in order to analyse its behaviour. In order to stabilize the entire system, linear Quadratic Regulator (LQR) control is used in such a way to set both PD and PID controls in position variables. A set simulation is performed to carry out the results for linear and non linear models. The LQR-PD and LQR-PID allow to move the plant's poles of UAV in the left half plane since without controller the systems is unstable. Simulations, LQR-PD and LQR-PID controllers are designed by using Matlab/Simulink. The simulations are performed to show how LQR tuned PD and PID controllers lead to zero the error of the position along Z earth direction, stop the rotation of Unmanned Aerial Vehicle (UAV) around body axes and stabilize the hexarotor.

Keywords: Hexarotor; LQR, PID and PD controller.

1 Introduction

A hexarotor is a kind of non-coaxial multi-rotor aircraft which can achieve vertical take-off and landing (VTOL). The flight attitude control of the hexarotor can be achieved only by adjusting the speed of the six rotors. Compared with the conventional rotor-type aircrafts, as no tail, hexarotors have a more compact structure. Six rotors' lifting force is more uniform than a single rotor, and thus the flight attitude is more stable. The take-off requirements of hexarotor are lower than those of a fixed-wing aircraft; moreover, it can hover and it has a better environmental adaptability [1]. As an important representative of multi-rotor aircrafts, the hexarotor has became a new aviation research frontier in the field of aviation and aircraft [2]. In the past few years, many research efforts have been done in this field, such as Mesicopter [3], X4 Flyer project of Australian National University [5], Draganflyer of Massachusetts Institute of Technology [6]. The movement of the hexarotor is generated by the lifting force which is provided by the motor driven propellers [7], [10]. By controlling the speed of the six rotors, vertical takeoff and landing, hovering and other movement can be achieved. Thus, a robust control system based on an efficient mathematical model can allow easily to maneuver and to manage the flight of the drone. The mathematical model derives

from Newton-Euler equations [8] and it is linearised by means of the theory of small perturbations. As control techniques concerns, several of them have been studied by different authors in order to improve the performance of this kind of system. For example, in [4] a control strategy based on backstepping procedure joint with Proportional Integrative Derivative (PID) technique is presented for regulating the dynamics of a quadrotor. On the other hand, the controller introduced in [12] stabilizes the attitude of a quadrotor by means of a compensation term and of a sliding mode term, while the authors of [11] describe an integrator backstepping control on attitude that stabilizes the drone around the equilibrium point.
The present paper compares two optimal control algorithms that combine a Proportional Derivative (PD) and a Proportional Integrative Derivative (PID) controller with a Linear-Quadratic Regulator (LQR). In detail, the first algorithm makes use of LQR in order to set PD parameters, as well as, in the second algorithm LQR tuned PID controller. For sake of simplicity, from now on, they will be referred to as LQR-PD and LQR-PID, respectively. Presented simulations show that the entire non linear system is stabilized with respect to a fixed setpoint for desired variables of interest.
Moreover, the performance of the LQR-PD algorithm and of the LQR-PID one have been compared, taking into account both the entire system and its linearisation around a hovering configuration. Disturbances introduced on altitude and angular velocity are controlled assuring the stabilization of the entire system.

2 Dynamic Model

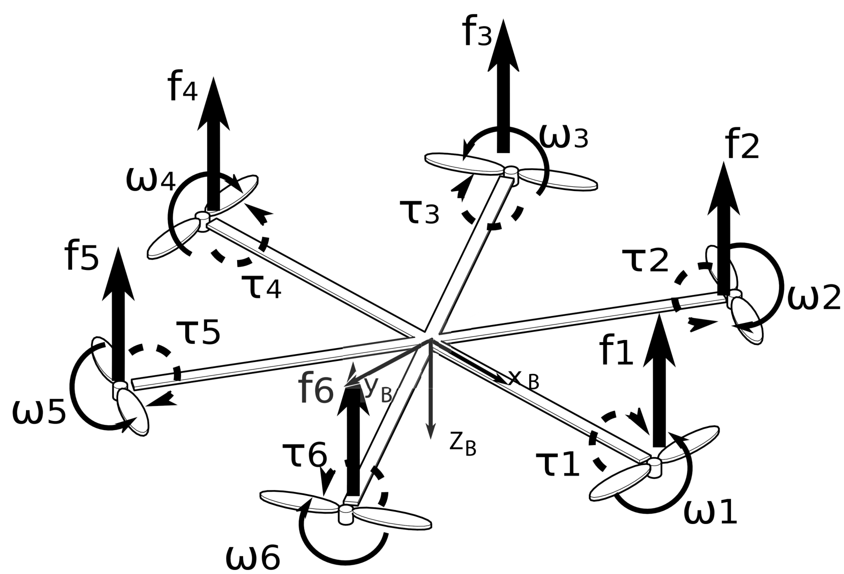

Figure 1: Scheme of the hexacopter.

This section deals with the mathematical model governing the motion of an unmanned aerial vehicle (UAV). This paper takes into account a hexacopter that is a hexagon shaped aerial vehicle, equipped with three pairs of fixed pitch propellers rotating in opposite direction. The scheme of the hexacopter is illustrated in Figure 1, where (x_B, y_B, z_B) represents the mobile frame attached to the body and located on the centre of gravity. The earth fixed frame is in located on the earth surface and the X, Y, and Z axes are directed to the North, East and down, respectively.

2.1 Non Linear Model

The Newton-Euler equations describe the complete dynamics of the hexacopter under the hypothesis of ridig body. As the translational component concerns, the total force acting on the UAV is the results of the total thrust $[0,\ 0,\ T]^T$ and the gravitational force $[0,\ 0,\ mg]^T$, in which m is the mass and g is the gravitational acceleration. On the other hand, the equation that governs the rotational component of the hexacopter dynamics states that the rate of change of the angular momentum balances the difference between the external torque $[\tau_\phi,\ \tau_\theta,\ \tau_\psi]^T$ and the gyroscopic forces Γ. Thus, the non linear dynamics of the hexacopter is here described in terms of Euler angles ϕ, θ and ψ, i.e.

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} T\left(\cos(\psi)\sin(\theta)\cos(\phi) + \sin(\psi)\sin(\phi)\right)/m \\ T\left(\sin(\psi)\sin(\theta)\cos(\theta) - \cos(\psi)\sin(\theta)\right)/m \\ g - T\cos(\theta)\cos(\phi)/m \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} (I_{yy} - I_{zz})\, q\, r/I_{xx} \\ (I_{zz} - I_{xx})\, p\, r/I_{yy} \\ (I_{xx} - I_{yy})\, p\, q/I_{zz} \end{bmatrix} - I_r \begin{bmatrix} q/I_{xx} \\ -p/I_{yy} \\ 0 \end{bmatrix} \omega_\Gamma + \begin{bmatrix} \tau_\phi/I_{xx} \\ \tau_\theta/I_{yy} \\ \tau_\psi/I_{zz} \end{bmatrix} \tag{2}$$

where $I_{xx},\ I_{yy},\ I_{zz}$ are the inertia moment with respect to $x,\ y$ and z axes, respectively, I_r is the moment of inertia with respect to the z axis of the six motor propeller assemblies, $\omega_\Gamma = \omega_1 - \omega_2 + \omega_3 - \omega_4 + \omega_5 - \omega_6$ being $\omega_i, i = 1,..,6$ the propeller angular velocities and $[p,\ q,\ r]^T$ is the angular rate vector depending on the Euler angles through the following transformation, [13],

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta\cos\phi \\ 0 & -\sin\phi & \cos\theta\cos\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

3 LQR Control and Linearization Model

Linear quadratic regulator is one of the modern control technique widely applied due to its optimality for linear time invariant system, such as

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BU} \tag{3}$$

where $\mathbf{X}$ is the state vector, $\mathbf{U}$ is the input one, $\mathbf{A}$ is the system matrix and $\mathbf{B}$ is the input matrix. The LQR approach, in the case of full state feedback control, is based on the minimization of the quadratic cost function

$$\mathbf{J} = \lim_{t \to +\infty} \int_0^t \left(\mathbf{X^T\ Q\ X} + \alpha \mathbf{U^T\ R\ U} \right) \tag{4}$$

in which $\mathbf{Q}$ and $\mathbf{R}$ can be chosen according to Bryson rule while $\alpha \in \mathbf{R}$ is a parameter chosen by trial and error approach.

In order to apply the LQR control, as in [9], the non linear system (1)-(2) is linearised around a hovering configuration $\mathbf{X_h}$, such that the angular velocity $[p, q, r]^T$ corresponds to Euler angle time derivative, i.e. $[p\ q\ r]^T = \left[\dot{\phi}\ \dot{\theta}\ \dot{\psi}\right]^T$.

Briefly, the dynamical system leads to

$$\begin{cases} \ddot{z} &=& g - T/m \\ \ddot{\phi} &=& \tau_\phi/I_{xx} \\ \ddot{\theta} &=& \tau_\theta/I_{yy} \\ \ddot{\psi} &=& \tau_\psi/I_{zz} \end{cases} \tag{5}$$

3.1 LQR-PD

The first algorithm introduced in this paper deals with a LQR control used to compute the parameters of the proportional and derivative control on altitude and Euler angles. For such reason the state vector can be written as $\mathbf{X} = \left[z\ \dot{z}\ \phi\ \theta\ \psi\ \dot{\phi}\ \dot{\theta}\ \dot{\psi}\right]^T$, while the input vector is given by $\mathbf{U} = [T\ \tau_\phi\ \tau_\theta\ \tau_\psi]$, according with the system (5). The system and input matrices $\mathbf{A} \in \mathbf{R}^{8\times 8}$ and $\mathbf{B} \in \mathbf{R}^{8\times 4}$ are sparse with

$$A_{1,2} = A_{3,6} = A_{4,7} = A_{5,8} = 1 \tag{6}$$

$$B_{2,1} = -1/m;\ \ B_{6,2} = 1/I_{xx};\ \ B_{7,3} = 1/I_{yy};\ \ B_{8,4} = 1/I_{zz}. \tag{7}$$

Thus, the linearised dynamical system assumes the following vectorial notation

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BU} + \mathbf{e_2}g \tag{8}$$

According with the LQR technique, the Input vector $\mathbf{U}$ depends on $\mathbf{X}$ through the coefficient matrix $\mathbf{K} \in \mathbf{R}^{4\times 8}$, that is

$$\mathbf{U} = \mathbf{U_0} - \mathbf{KX} \tag{9}$$

being $\mathbf{U_0} = [mg,\ 0,\ 0,\ 0]^T$ the initial condition of inputs quantities while $\mathbf{K}$ is expressed as $\mathbf{K} = \mathbf{R^{-1}BS}$ according to the solution of the Lurie-Riccati equation [14],

$$\mathbf{A^TS} + \mathbf{SA} - \mathbf{SBR^1B^TS} + \mathbf{Q} = 0 \tag{10}$$

Note that,

$$\mathbf{KX} = \mathbf{K_P}\left[z\ 0\ \phi\ \theta\ \psi\ 0\ 0\ 0\right]^T + \mathbf{K_D}\left[0\ \dot{z}\ 0\ 0\ 0\ \dot{\phi}\ \dot{\theta}\ \dot{\psi}\right]^T \tag{11}$$

with $\mathbf{K_P}$, $\mathbf{K_D} \in \mathbf{R}^{4\times 8}$ suitable matrices. In such way, the entries of the gained $\mathbf{K}$, depending on $\mathbf{A}$, $\mathbf{B}$, $\mathbf{Q}$ and $\mathbf{R}$ are chosen as the optimal parameters of PD controller. So, the system (8) leads to

$$\dot{\mathbf{X}} = (\mathbf{A} - \mathbf{BK})\,\mathbf{X} + \mathbf{BU_0} + \mathbf{e_2}g \tag{12}$$

3.2 LQR-PID

The second algorithm investigated is based on LQR approach tuning a proportional, integrative and derivative control on z, ϕ, θ and ψ. For this purpose, the state vector becomes

$$\tilde{\mathbf{X}} = \left[\int_0^t z(s)ds\ \ \int_0^t \phi(s)ds\ \ \int_0^t \theta(s)ds\ \ \int_0^t \psi(s)ds\ \ z\ \ \dot{z}\ \ \phi\ \ \theta\ \ \psi\ \ \dot{\phi}\ \ \dot{\theta}\ \ \dot{\psi}\right]^T$$

while the matrices $\tilde{\mathbf{A}} \in \mathbf{R}^{12\times 12}$ e $\tilde{\mathbf{B}} \in \mathbf{R}^{12\times 4}$ are still sparse with

$$\tilde{A}_{1,5} = \tilde{A}_{2,7} = \tilde{A}_{3,8} = \tilde{A}_{4,9} = \tilde{A}_{5,6} = \tilde{A}_{7,10} = \tilde{A}_{8,11} = \tilde{A}_{9,12} = 1$$

$$\tilde{B}_{6,1} = -1/m;\ \ \tilde{B}_{10,2} = 1/I_{xx};\ \ \tilde{B}_{11,3} = 1/I_{yy};\ \ \tilde{B}_{12,4} = 1/I_{zz}.$$

Thus, under the assumption $\mathbf{U} = \mathbf{U_0} - \tilde{\mathbf{K}}\tilde{\mathbf{X}}$, the linear system (9) can be written as

$$\dot{\tilde{\mathbf{X}}} = \left(\tilde{\mathbf{A}} - \tilde{\mathbf{B}}\tilde{\mathbf{K}}\right)\tilde{\mathbf{X}} + \tilde{\mathbf{B}}\mathbf{U_0} + \mathbf{e_6}g \tag{13}$$

Instead, in this case,

$$\begin{array}{lllllllllllllll}
\tilde{\mathbf{K}}\tilde{\mathbf{X}} & = \tilde{\mathbf{K_I}} & [& \int_0^t z(s)ds & \int_0^t \phi(s)ds & \int_0^t \theta(s)ds & \int_0^t \psi(s)ds & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 &]^T + \\
 & +\tilde{\mathbf{K_P}} & [& 0 & 0 & 0 & 0 & z & 0 & \phi & \theta & \psi & 0 & 0 & 0 &]^T + \\
 & +\tilde{\mathbf{K_D}} & [& 0 & 0 & 0 & 0 & 0 & \dot{z} & 0 & 0 & 0 & \dot{\phi} & \dot{\theta} & \dot{\psi} &]^T
\end{array} \tag{14}$$

with appropriate $\tilde{\mathbf{K_I}},\ \tilde{\mathbf{K_P}},\ \tilde{\mathbf{K_D}} \in \mathbf{R}^{4\times 12}$. Here, again, the entries of $\mathbf{K}$ matrix indicate the coefficients of the PID control.

4 Numerical Results

In this section, the results obtained by the implementation of the two investigated algorithms are presented. In details, the LQR-PD and LQR-PID approaches have been tested for the stabilization of the linear model around the hovering position, and, then, they have been applied to the entire non linear system. Examples show the reaction of both systems to impulse introduced on $\dot{z}$, $\dot{\phi}$, $\dot{\theta}$, $\dot{\psi}$, taking into account a hexacopter with mass $m = 16.296\ \ kg$, $I_{xx} = I_{yy} = 0.9458\ \ kg\, m^2, I_{zz} = 1.7055\ \ kg\, m^2$, $I_r = 0.1380\ \ kg\, m^2$.

4.1 LQR-PD Controller

In numerical tests concerning the LQR-PD control, the matrices $\mathbf{Q}$ and $\mathbf{R}$, according with the Bryson rule, are

$$\mathbf{Q} = diag[0.5,\ 0.0001,\ 1.22,\ 1.22,\ 1.22,\ 0.0001,\ 0.0001,\ 0.0001] \times 10^4$$

$$\mathbf{R} = diag[0.5,\ 0.5,\ 0.5,\ 0.5] \times 10^{-4},$$

with $\alpha = 1$. Moreover, the initial condition is

$$\mathbf{X_0} = \mathbf{X_h} = [-1,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0]\,.$$

A disturbance on $\dot{z}$ is introduced after 2 seconds, like an impulse with amplitude 10 m/s and pulse width 0.1 s, as shown in Figure 2, where the altitude of the hexacopter under the influence of the assigned disturbance are also reported for both the non-linear and linearised models. As expected, the LQR-PD controller leads to the stabilization of the hexacopter around the hovering configuration.

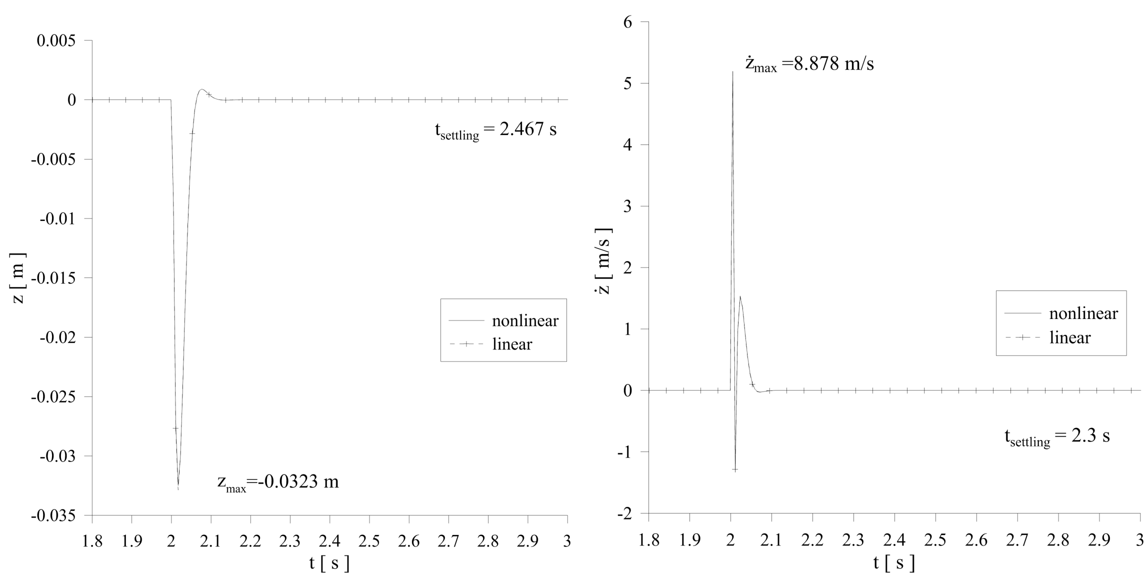

Figure 2: Altitude and translational velocity along z direction, under the disturbance of $\dot{z}$ obtained by LQR-PD scheme.

Then, a similar disturbance, an impulse with amplitude 1 rad/s and pulse width 0.1 s, is applied to $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$. Figures 3, 4 and 5 illustrate the disturbance on the time derivative of the Euler angles that affect the torques and the trend of the Euler angles that eventually tend to zero. In other words the drone is stabilized around hovering altitude in less than $0.5\,s$ while its rotations with respect to the body frame nullifies with characteristic settling times less then $0.2\,s$.

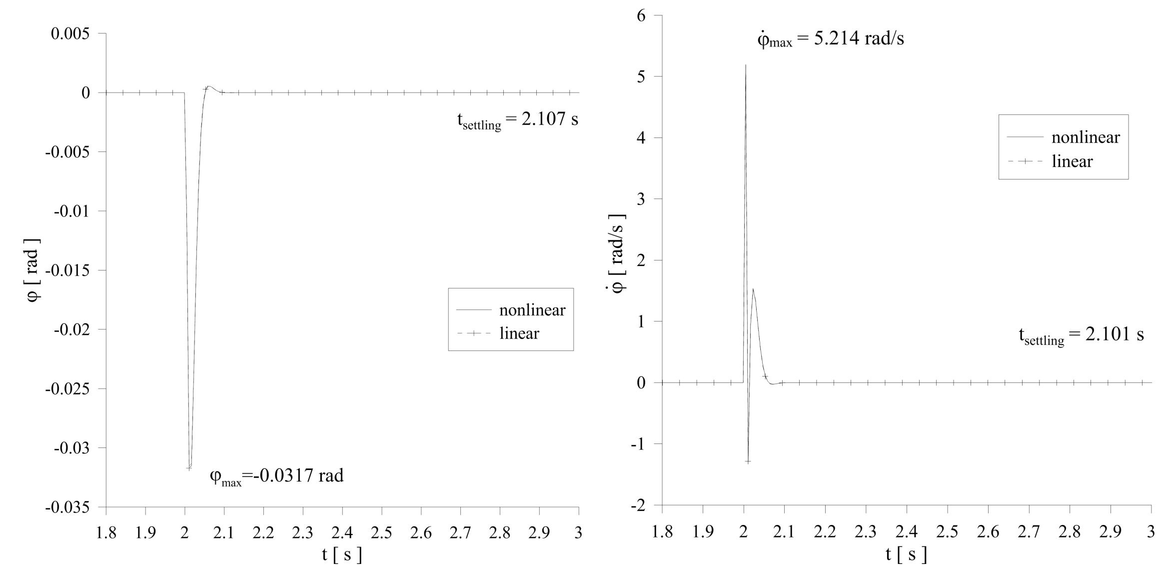

Figure 3: Roll angle ϕ and its time derivative $\dot{\phi}$ obtained by LQR-PD scheme.

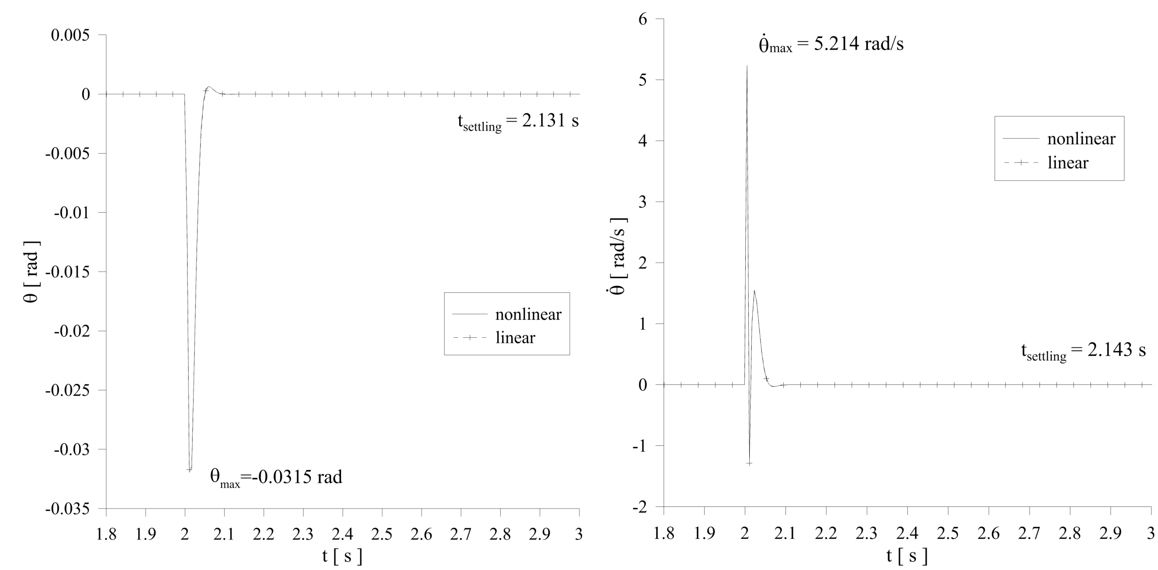

Figure 4: Pitch angle θ and its time derivative $\dot{\theta}$ obtained by LQR-PD scheme.

4.2 LQR-PID Controller

On the other hand, if LQR-PID is taken into account,

$$\mathbf{Q} = diag[10^{-8},\ 10^{-6},\ 10^{-6},\ 10^{-6},\ 5 \times 10^{3},\ 0.5,\ 10^{4},\ 10^{4},\ 10^{4},\ 0.5,\ 0.5,\ 0.5]$$

R keeps unchanged and $\alpha = 0.4150$, chosen by trial and error approach. The dynamical system are solved with initial condition

$$\tilde{\mathbf{X}}_{\mathbf{0}} = [0,\ 0,\ 0,\ 0,\ -1,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0] \qquad (15)$$

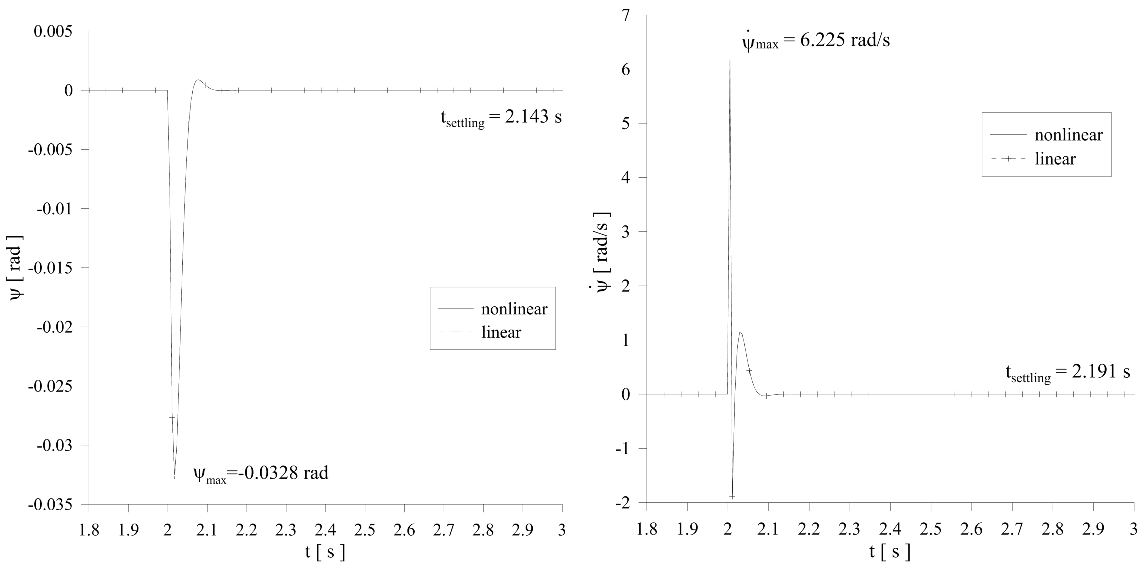

Figure 5: Yaw angle ψ and its time derivative $\dot{\psi}$ obtained by LQR-PD scheme.

As in the previous section, a disturbance on $\dot{z}$ is introduced after 2 seconds, like an impulse with amplitude 10 m/s and pulse width 0.1 s, as shown in Figure 6. Moreover the altitude of the hexacopter under the influence of the assigned disturbance is presented in Figure 6, where, thanks to LQR-PID controller, the stabilization of the hexacopter around the hovering configuration is shown. After 2 seconds,

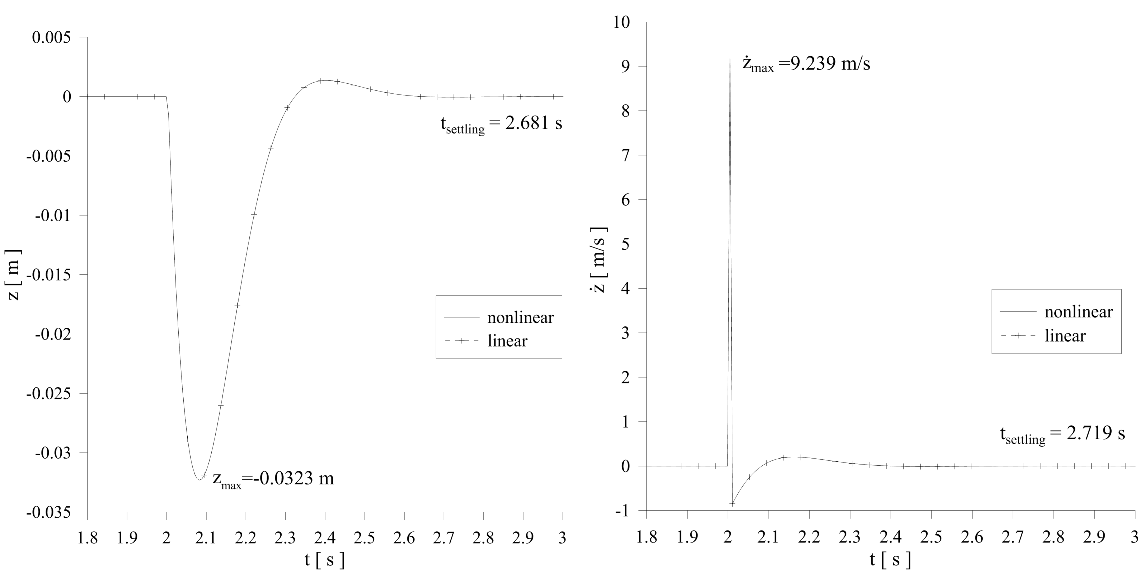

Figure 6: Altitude and translational velocity along z direction, under the disturbance of $\dot{z}$ obtained by LQR-PID scheme.

an impulse with amplitude 1 rad/s and pulse width 0.1 s, is applied to $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$ that perturbs the angular velocities as presented in Figure 7, 8 and 9. However, the hovering attitude configuration is recovered in less that 0.2 seconds, comparable with the LQR tuned PD scheme, but the damping process involved in the altitude stabilization appears longer than the PD one for the given parameters. Moreover, by taking into account the variables maximum deviation with respect to the hovering initial state it stems that no appreciable discrepancies are obtained in the position quantities evaluated through both PD and PID control schemes. On the other hand, it can be concluded that the LQR-PD approach gives lower peaks of the variables time rate with respect the LQR-PID ones.

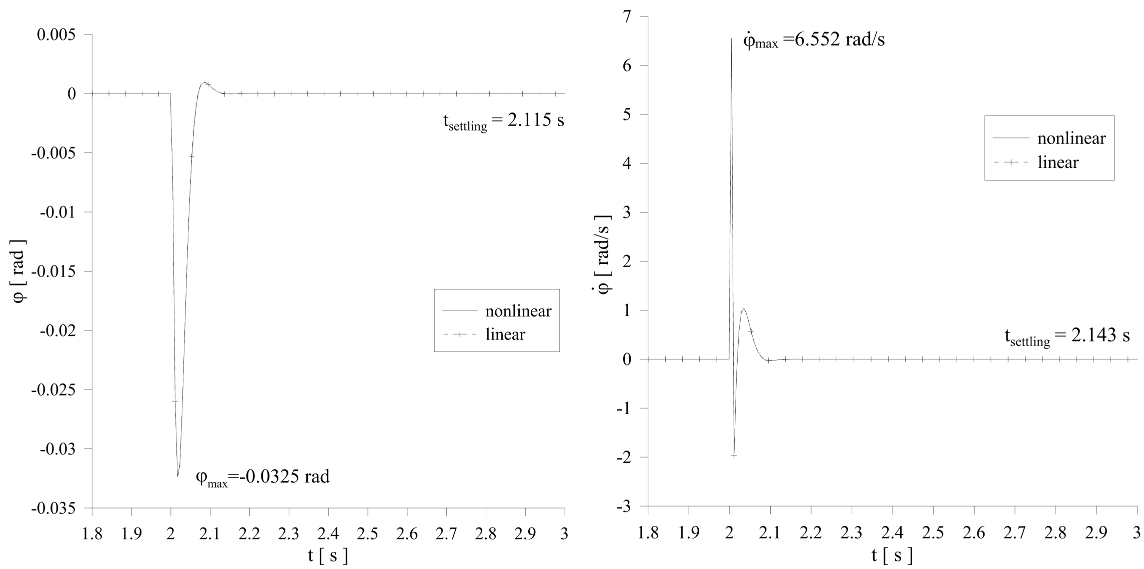

Figure 7: Roll angle ϕ and its time derivative $\dot{\phi}$ obtained by LQR-PID scheme.

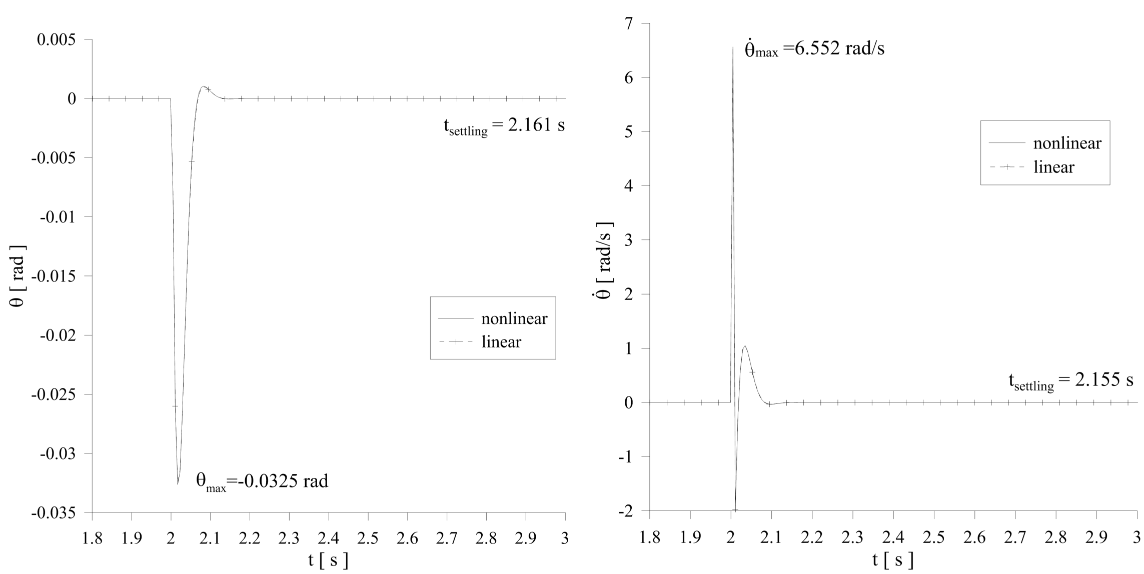

Figure 8: Pitch angle θ and its time derivative $\dot{\theta}$ obtained by LQR-PID scheme.

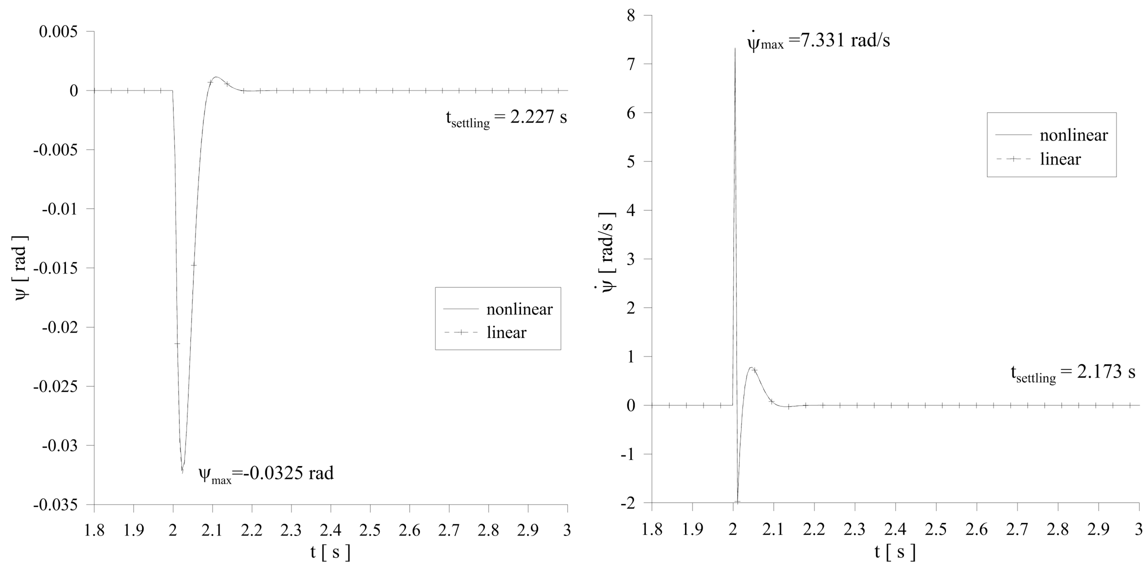

Figure 9: Yaw angle ψ and its time derivative $\dot{\psi}$ obtained by LQR-PID scheme.

5 Conclusion

In this paper the dynamical behaviour of a hexarotor has been analysed with the aim of stabilizing it around the hovering configuration even in presence of external disturbances. For this reason the LQR control has been implemented for tuning both PD and PID parameters. The obtained LQR-PD and LQR-PID controllers have been tested by comparing the response to an impulse disturbances of the nonlinear dynamical system with the response of the linearised one. No appreciable discrepancies have been evidenced between the linear and nonlinear models for the given disturbances. Results show that the presented methods stabilize the perturbed system around the equilibrium position, in about half a second.

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