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Preface

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This special journal edition highlights some of the recent work on two-point boundary value problems and geometric integration that has been carried out by groups in Italy and the United Kingdom. In the area of two-point boundary value problems these groups are at the forefront of currect research and they have developed several powerful algorithms for the numerical solution of such problems. The papers on geometric integration make a contribution to a very important and rapidly expanding research area which has many important practical applications.

The first paper by Capper and Moore (CM) is concerned with the derivation of high order mono-implicit Runge-Kutta formulae. Such formulae have been widely used in a deferred correction framework (see references [3], [5], [7], [14] in CM) and have proved to be very efficient for the numerical solution of general first order systems of nonlinear two-point boundary value problems. In particular the code TWPBVP (see ref [9] of Cash and Mazzia) is based on mono implicit Runge-Kutta formulae of orders 4,6 and 8. In the Capper/Moore paper MIRK formulae of order 10 and 12 are derived. Also derived in this paper are high order interpolants which allow continuous solutions to be defined. These very high order Runge-Kutta methods are analysed in detail and in particular it is shown that the numerous error coefficients appearing in the expressions for the local truncation error of the two formulae are extremely small. These new methods would be expected to be very efficient when a high order of accuracy is requested and extensive numerical experiments demonstrate that this is indeed the case. The next paper by Capper, Cash and Moore (CCM) concerns the numerical solution of second order two point boundary value problems having the special form $y'' = f(x, y)$ or $y'' = f(x, y, y')$. Many two-point BVPs appear in this form and a lot of important structure is lost if they are converted to first order form. Indeed tremendous savings in computational effort can be made if we retain this second order structure when deriving numerical algorithms. In the Capper/ Cash/ Moore paper formulae of order 6 and 8 are derived for second order equations and the great efficiency of these new formulae is demonstrated by numerical experiments. The third paper is by Cash and Mazzia (CM). This paper is concerned with the construction of mesh selection algorithms which are based on the conditioning of the problem as well as on the accuracy of the solution. In the past, grid choosing algorithms have typically been based on an attempt to keep a local error estimate less than a prescribed local maximum and no account was taken of the conditioning of the problem. The first papers to show that conditioning is important in mesh selection and to derive algorithms based on conditioning and local accuracy were due to Brugnano, Trigiante and Mazzia (see CM references [5],[13]). In (CM ref [8]) Cash and Mazzia derived a mesh selection algorithm based on conditioning and accuracy for use with the deferred correction code TWPBVP.f. The analysis for this is relatively straightforward because the deferred corrections are explicit. In the Cash/Mazzia paper these ideas are extended to deferred correction schemes based on Lobatto formulae. This algorithm is difficult to analyse because the deferred corrections are implicit. However a careful analysis of the explicit case does allow an efficient mesh choosing algorithm to be derived for the implicit case and extensive numerical results are given to show that the new algorithm is considerably more efficient than the existing one. The new algorithms are available on the web pages of the authors. The next paper is by Cash, Surmati, Abdulla and Vieira (CSAV). This paper deals with the derivation of interpolants for use in the numerical solution of boundary value problems. It is pointed out that most codes for solving boundary value problems do compute interpolants as a matter of course but the way in which they do it tends to be different from code to code. For example, the well known code COLSYS seeks to provide a continuous solution from the outset, the code MIRKDC extends the class of MIRK formulae used to the continuous case whereas the deferred correction codes compute error estimates aposteriori. In CSAV attention is paid to the problem of deriving interpolant for the Lobatto code TWPBVPL.f. It is shown that explicit interpolants tend to perform very poorly when used with this code. This is of course due to the relatively poor stability of the interpolants. It is shown that a posteriori implicit interpolants can be computed using very little extra computational effort and numerical results are given to show that these implicit interpolants perform very well in practice. The next paper by Mazzia, Sestini and Trigiante (MST) considers the numerical solution of two-point boundary value problems using $B-$splines. To do this they define a new class of methods, known as BS methods, which are based on $B-$splines and which are a special class of linear multistep methods. The formulae which they derive are unconditionally convergent and have excellent stability properties when used in the well known boundary value method framework. These new methods are analysed in detail and a method based on non-uniform meshes and having a continuous solution is derived. The efficiency of the new method is demonstrated by means of numerical results when it is applied to some difficult singular perturbation problems. The next paper is by Amodio, Gladwell and Romanazzi and is concerned with the numerical solution of general bordered ABD linear systems using a method which is a generalisation of the well known cyclic reduction algorithm. Systems of algebraic equations of this type typically arise from the solution of nonlinear boundary value problems having non-separated boundary conditions. It is particularly important that structured systems of algebraic equations are solved as efficiently as possible since this can be a large part of the overall computational cost. The new algorithm is shown to be more effective than several well known existing algorithms and in particular the improved efficiency compared to COLNEW is shown. The next paper by Cash and Girdlestone is concerned with the numerical solution of ρ reversible initial value problems. It is possible to derive fixed stepsize numerical integrators which are very efficient for the numerical solution of a large class of such problems. However for some problems it is important to be able to use variable stepsize. However if a standard method for varying the step is used this can often completely destroy the good behaviour of the underlying method. Instead, if a variable stepsize of integration is to be used it is necessary to choose a special symmetric step choosing algorithm such as the one suggested by Stoffer. In the Cash/Girdlestone paper some high order symmetric Runge-Kutta-Nystrom formulae with special step choosing algorithms are developed and their efficiency compared to standard integration formulae is demonstrated by means of some numerical examples. The final paper is by Iavenaro and Trigiante and it is concerned with the numerical solution of Hamiltonian problems using symmetric integration formulae. There is a great deal of numerical evidence to indicate that symmetric formulae perform very well when applied to dynamical systems that have a first integral. The standard way for showing this is via a backward error analysis. In the present paper the following question is posed:'Is it possible after applying a given integration method to a continuous Hamiltonian system to identify a quantity which is exactly preserved by the numerical method and which can be interpreted as the discrete energy'?. This problem is examined in some detail and it is shown how such a discrete energy function can often be identified.

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Jeff Cash is professor of Numerical Analysis at Imperial College London. He graduated from Imperial College with a first class degree in 1969 and studied for a PhD under the guidance of Dr. J. C. P. Miller at Cambridge. The title of his thesis was 'Numerical methods for differential and difference equations'. He left Cambridge to take up a lectureship at Imperial College and has been there ever since!!. He has published numerous papers in the field of Numerical Analysis and is particularly interested in the numerical solution of initial and boundary value problems for ODEs, in numerical methods for the solution of time dependent partial differential equations and in linear algebra. He is an honorary fellow of the ESCMSE.