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Painlevé analysis and exact solutions of generalized nonlinear Vakhnenko-Parkes equation

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Abstract: In this paper it is investigated the generalized Vakhnenko-Parkes equation describing the propagation of short-wavelength disturbances in relaxing media with amplitude-dependent wave velocity. As the Cauchy problem for this equation proves intractable to the inverse scattering transform method, the analysis employs traveling wave variables. To study the integrability of an ordinary differential equation obtained by moving to traveling wave variables, the Painlevé test is used. A general solution to this ordinary differential equation, written in terms of quadrature, is obtained. The dependence of the solution on parameters of the equation is investigated. Based on the analysis, explicit exact solutions in traveling wave variables are found. It is shown that the equation has periodic solutions expressed through Jacobi elliptic functions, as well as an singular analytical solution in explicit form. Obtained exact solutions can be used as test functions in the analysis of results of numerical modeling of processes in relaxing media described by equations of the Vakhnenko-Parkes type.

Keywords: Vakhnenko-Parkes equation, Painlevé analysis, exact solutions, solutions in quadratures, traveling wave solutions, Jacobi elliptic functions

Mathematics Subject Classification: 35Q51, 34M55, 35C07

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1 Introduction

In this paper, the equation

$$u u_{xxt} + \alpha u_x u_{xt} + (\beta u^2 + \gamma u^3) u_t = 0. \tag{1}$$

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for the real function $u = u(x,t)$ is considered. In equation (1) parameters α, β, γ are real nonzero numbers. This equation is a generalization of the nonlinear Vakhnenko-Parkes equation [1–13]

$$uu_{xxt} - u_x u_{xt} + u^2 u_t = 0 \tag{2}$$

and the modified Vakhnenko-Parkes equation [14–18]

$$uu_{xxt} - u_x u_{xt} + u^3 u_t = 0. \tag{3}$$

The equation (2) is used to describe the propagation of short-wave disturbances in relaxing media in case when the amplitude of oscillations depends on the wave propagation velocity. The motivation for considering this kind of equations is the fact that the dispersion relation for the classical Korteweg-de Vries (KdV) equation is overly restrictive in describing the propagation of short-wave disturbances. In addition, despite the fact that the KdV equation allows for solitary and periodic solutions that are not provided for by shallow water theory, it does not describe the observed peaking of wave crests. In turn, the equation (2) takes into account both of these effects.

Moreover, the equation (1) is one of equations of a family of partial differential equations

$$uu_{xxt} + \alpha u_x u_{xt} + (\beta u^{2n} + \gamma u^{3n}) u_t = 0, \tag{4}$$

where $n \neq \frac{1}{3}$. This family of equations is considered in the article [19], where it is reduced to a dynamical system using traveling wave variables. Further research is conducted on the basis of the qualitative theory of differential equations. Let us rewrite the equation (1) using traveling wave variables:

$$u = \phi(\xi), \quad \xi = x - C_0 t, \tag{5}$$

where C_0 is the wave propagation velocity. Substituting the variables (5) into the equation (1) and multiplying it by ϕ_ξ, we integrate it once by ξ. As a result, we obtain the following nonlinear ordinary differential equation of the second order:

$$\phi\phi_{\xi\xi} + \frac{\alpha - 1}{2}\phi_\xi^2 + \frac{\gamma}{4}\phi^4 + \frac{\beta}{3}\phi^3 + g = 0, \tag{6}$$

where g is the integration constant. The obtained equation is a special case of the equation considered in [20] of B. Gambier:

$$ayy_{zz} + by_z^2 + c_4 y^4 + c_3 y^3 + c_2 y^2 + c_1 y + c_0 = 0, \tag{7}$$

where $y = y(z)$. It is known that the solution to the equation (7) is expressed in terms of elliptic functions in following cases:

1. $a = 1$, $b = -1$, $c_1 = c_2 = 0$;
2. $a = 2$, $b = -1$, $c_1 = c_2 = 0$;
3. $a = -2b$, $c_1 = 0$.

In [19] some exact solutions of the equation (6) are also presented for the cases $\alpha = -1$, $\alpha > 1$, depending on the values of γ and the integration constant in the first integrals of the dynamical system.

The purpose of this article is to study the integrability of the equation (6) and to construct its analytical solutions. The integrability investigation applying the Painlevé test in the form of Kovalevskaya algorithm is described in section 2. Methods for finding the general solution to the equation (6) in terms of quadrature are briefly explained in section 3. Based on the obtained general solution, in section 4 several forms of exact solutions to the equation (6) are shown [21].

Currently, there are many mathematical models describing various processes in the applied mathematics, pure and technical physics, chemistry, biology, economics etc. in terms of problems for differential equations. In the context of the numerical investigation of mentioned problems, the question of the results verification is crucial. To estimate the adequacy of a numerical solution to the problem, it is important to compare it with an analytical solution. Thus, aside from a potential value of this work in the domain of the general theory of differential equations, obtained results, in particular, can be used as test functions for the data verification of the numerical modeling of the wave propagation in relaxing media and rotating fluid described by equations of the Vakhnenko-Parkes type.

2 Painlevé analysis of the equation (6)

To study the equation (6) for passing the Painlevé test below, we apply a S.V. Kovalevskaya algorithm, improved in [22, 23]. This technique can give the opportunity of the general solution existence [24–29]. Within the framework of this algorithm, we present the solution to the equation (6) in the form of a Laurent series expansion near a singular point $\xi = \xi_0$:

$$\phi(\xi) = \sum_{j=0}^{\infty} a_j (\xi - \xi_0)^{j-p}, \tag{8}$$

where p is a solution pole order. It is known that the Kovalevskaya algorithm consists of three steps: finding the pole order of the equation under consideration, determining Fuchs indices (also known as resonances), and substituting the expansion into the equation in order to find coefficients for powers of ξ.

At the first step of the Kovalevskaya algorithm, let us consider the equation with leading terms for the equation (6) in the form

$$\phi\phi_{\xi\xi} + \frac{\alpha-1}{2}\phi_\xi^2 + \frac{\gamma}{4}\phi^4 = 0. \tag{9}$$

Note that the equation (6) is autonomous, hence it is possible to omit the singular point ξ_0 at first and second steps of the Painlevé test. Therefore, to determine the pole order of the solution to the equation (6), we substitute the expression

$$\phi(\xi) = \frac{a_0}{z^p} \tag{10}$$

into the equation (9). By equating expressions with the same degrees of ξ in the equation (9) to find the value of p and then calculating the value of a_0, we obtain two branches of solutions to the equation (6) in the form

$$(a_0, p) = \left(\pm\sqrt{-\frac{2(\alpha+3)}{\gamma}}, 1 \right). \tag{11}$$

Thus, the solution to the equation (6) has the first pole order. To simplify further transformations during the Painlevé test, we introduce the next substitution in the equation (6):

$$\gamma = -2(\alpha+3), \tag{12}$$

then it takes the form

$$\phi\phi_{\xi\xi} + \frac{\alpha-1}{2}\phi_\xi^2 - \frac{\alpha+3}{2}\phi^4 + \frac{\beta}{3}\phi^3 + g = 0. \tag{13}$$

α	Equation form	$a_j^{(\alpha)}$	$K_{\alpha+3}$
-3	$\phi\phi_{\xi\xi} - 2\phi_\xi^2 + \dfrac{\beta}{3}\phi^3 + g = 0$	undefined	$\frac{\beta(a_0^{(-3)})^2}{3}$
-2	$\phi\phi_{\xi\xi} - \frac{3}{2}\phi_\xi^2 - \frac{1}{2}\phi^4 + \frac{\beta}{3}\phi^3 + g = 0$	$a_0^{(-2)} = \pm 1$	$\pm\frac{\beta}{3}$
-1	$\phi\phi_{\xi\xi} - \phi_\xi^2 - \phi^4 + \frac{\beta}{3}\phi^3 + g = 0$	$a_0^{(-1)} = \pm 1,\ a_1^{(-1)} = \frac{\beta}{6}$	0
1	$\phi\phi_{\xi\xi} - 2\phi^4 + \frac{\beta}{3}\phi^3 + g = 0$	$a_0^{(1)} = \pm 1,\ a_1^{(1)} = \frac{\beta}{18},$ $a_2^{(1)} = \pm\frac{\beta^2}{324},\ a_3^{(1)} = \frac{\beta^3}{5832}$	g
2	$\phi\phi_{\xi\xi} + \frac{1}{2}\phi_\xi^2 - \frac{5}{2}\phi^4 + \frac{\beta}{3}\phi^3 + g = 0$	$a_0^{(2)} = \pm 1,\ a_1^{(2)} = \frac{\beta}{24},\ a_2^{(2)} = \pm\frac{\beta^2}{576},$ $a_3^{(2)} = \frac{\beta^3}{13824},\ a_4^{(2)} = \pm\left(\frac{g}{5} + \frac{\beta^4}{331776}\right)$	0
3	$\phi\phi_{\xi\xi} + \phi_\xi^2 - 3\phi^4 + \frac{\beta}{3}\phi^3 + g = 0$	$a_0^{(3)} = \pm 1,\ a_1^{(3)} = \frac{\beta}{30},\ a_2^{(3)} = \pm\frac{\beta^2}{900},$ $a_3^{(3)} = \frac{\beta^3}{27000},\ a_4^{(3)} = \pm\left(\frac{g}{10} + \frac{\beta^4}{810000}\right),$ $a_5^{(3)} = \frac{\beta^5}{24300000}$	0
4	$\phi\phi_{\xi\xi} + \frac{3}{2}\phi_\xi^2 - \frac{7}{2}\phi^4 + \frac{\beta}{3}\phi^3 + g = 0$	$a_0^{(4)} = \pm 1,\ a_1^{(4)} = \frac{\beta}{36},\ a_2^{(4)} = \pm\frac{\beta^2}{1296},$ $a_3^{(4)} = \frac{\beta^3}{46656},\ a_4^{(4)} = \pm\left(\frac{g}{15} + \frac{\beta^4}{1679616}\right),$ $a_5^{(4)} = \frac{\beta^5}{60466176},\ a_6^{(4)} = \pm\left(\frac{\beta^6}{2176782336} + \frac{\beta^2 g}{45360}\right)$	0
5	$\phi\phi_{\xi\xi} + 2\phi_\xi^2 - 4\phi^4 + \frac{\beta}{3}\phi^3 + g = 0$	$a_0^{(5)} = \pm 1,\ a_1^{(5)} = \frac{\beta}{42},\ a_2^{(5)} = \pm\frac{\beta^2}{1764},$ $a_3^{(5)} = \frac{\beta^3}{74088},\ a_4^{(5)} = \pm\left(\frac{g}{20} + \frac{\beta^4}{3111696}\right),$ $a_5^{(5)} = \frac{\beta^5}{130691232},$ $a_6^{(5)} = \pm\left(\frac{\beta^6}{5489031744} + \frac{\beta^2 g}{82320}\right),$ $a_7^{(5)} = \frac{\beta^7}{230539333248} + \frac{\beta^3 g}{2963520}$	0

Table 1: The results of the Painlevé analysis of equation (6) for values $\alpha = -3;\ -2;\ -1;\ 1;\ 2;\ 3;\ 4;\ 5$.

In this case, the equation with the leading terms (9) is rewritten as

$$\phi\phi_{\xi\xi} + \frac{\alpha-1}{2}\phi_\xi^2 - \frac{\alpha+3}{2}\phi^4 = 0. \tag{14}$$

Taking into account introduced substitutions, conditions for the solution branches are written as

$$(a_0, p) = (\pm 1, 1). \tag{15}$$

At the second step of the Painlevé test, let us substitute the expression

$$\phi(\xi) = \frac{a_0}{\xi} + a_j \xi^{j-1} \tag{16}$$

to the equation with leading terms (14). Calculating the value of the coefficient of the first power of a_j for both branches (15), we have Fuchs indices $j = -1,\ j = \alpha + 3$. The equation passes the second step of the Painlevé test if Fuchs indices are natural pairwise distinct numbers excepting -1. Thus, in the case of $\alpha \in \mathbb{N} \cup \{-3; -2; -1\}$ the equation (6) passes the Painlevé test at the second step.

At the third step, the initial equation is examined at values of $\alpha = -3;\ -2;\ -1;\ 1;\ 2;\ 3;\ 4;\ 5$.

The decomposition of the solution into Laurent series is sought in the form

$$\phi(\xi) = \sum_{j=0}^{\alpha+3} a_j^{(\alpha)} \xi^{j-1}, \tag{17}$$

where $a_j^{(\alpha)}$ are coefficients of this decomposition depending on the value of α. Results of these studies are shown in table 1. Here, $K_{\alpha+3}$ represents the value of the coefficient at the lowest power of ξ, which is included in the equation when the expansion is substituted. The equation (6) would pass the Painlevé test if all values of $K_{\alpha+3}$ should be zero. It is visible from the table 1 that the equation (6) has the Painlevé property in cases of $\alpha = -1$, $\alpha = 2$, $\alpha = 3$, $\alpha = 4$ and $\alpha = 5$.

3 General solution to the equation (6)

To obtain the general solution of the equation (6), we rewrite its first two terms applying a technique used, for example, in [30]:

$$\phi\phi_{\xi\xi} + \frac{\alpha-1}{2}\phi_\xi^2 = \frac{2}{\alpha+1}\phi^{\frac{3-\alpha}{2}}(\phi^{\frac{\alpha+1}{2}})_{\xi\xi}. \tag{18}$$

Using the substitution $z(\phi) = \phi^{\frac{\alpha+1}{2}}$ allows us to find the first integral of the equation (6) and separate its variables. As a result, the general solution to this equation is written in terms of quadrature as follows:

$$\xi = \int \frac{d\phi}{\sqrt{-\frac{\gamma}{2(\alpha+3)}\phi^4 - \frac{2\beta}{3(\alpha+2)}\phi^3 - \frac{8C_1}{(\alpha+1)^2}\phi^{1-\alpha} - \frac{2g}{\alpha-1}}}, \tag{19}$$

where C_1 is an integration constant. It can be seen that the solutions of the equation (6) for $\alpha = -1$, $\alpha = 1$, $\alpha = -2$, $\alpha = -3$ cannot be expressed using the resulting quadrature. Furthermore, according to the results of the Painlevé analysis, there is no general solution to the equation (6) for the last three values of α.

4 Exact solutions to the equation (6)

With the appropriate values of the integration constants C_1 and g in the quadrature (19), exact solutions to the equation (6) can be found, as with other values of α. These cases are briefly outlined below.

Case $\alpha = -1$. The value of the integral (19) is undefined. By sequential substitutions of $\phi_\xi = w$ and $w = \sqrt{q(\phi)}$ in the equation (6), we have the following exact solution:

$$\phi(\xi) = \lambda_1 + \frac{\lambda_1\lambda_2 - \lambda_2\lambda_3 + \lambda_1\lambda_3 - \lambda_1^2}{(\lambda_3 - \lambda_2)\,\mathrm{sn}^2\left(\frac{1}{4}\sqrt{\gamma(\lambda_2-\lambda_4)(\lambda_1-\lambda_3)}(\xi - C_2); k\right)}, \tag{20}$$

where C_2 is an integration constant, λ_1, λ_2, λ_3, λ_4 are real roots of the polinomyal in the denominator of the expression (19), $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$, $k^2 = \frac{(\lambda_1-\lambda_4)(\lambda_1-\lambda_3)}{(\lambda_2-\lambda_3)(\lambda_2-\lambda_4)}$,

$$\begin{aligned} \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 &= -\frac{8\beta}{3\gamma} \\ \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4 &= -\frac{4C_2}{\gamma}, \\ \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4 &= 0, \\ \lambda_1\lambda_2\lambda_3\lambda_4 &= -\frac{4g}{\gamma}. \end{aligned} \tag{21}$$

Taking into account limits for roots λ_1, λ_2, λ_3, λ_4, it is evident that the obtained solution (20) represents a real function within the framework of the problem in the case of $\gamma > 0$. Solutions of a similar type also arise in cases of $C_1 = 0$ and $g = 0$, $\alpha = 1$ under appropriate constraints on parameters of the Jacobi elliptic sine.

Case $C_1 = g = 0$. In this case, an explicit power-law solution is obtained from quadrature (19), given by the expression

$$\phi(\xi) = -\frac{1}{\frac{\beta}{6(\alpha+2)}(\xi + C_3)^2 + \frac{3\gamma(\alpha+2)}{4\beta(\alpha+3)}} \tag{22}$$

It is visible that the obtained solution (22) is nontrivial in cases $\alpha \neq -2$, $\alpha \neq -3$ and has a singularity at points

$$\xi = \pm\frac{3(\alpha+2)}{\beta}\sqrt{-\frac{\gamma}{2(\alpha+3)}} - C_3. \tag{23}$$

Conclusion

In this paper, the generalized nonlinear Vakhnenko-Parkes equation is studied. Using traveling wave variables, this equation is reduced to an ordinary differential equation. A Painlevé analysis of the obtained equation was carried out. It was found that the equation passes the Painlevé test in cases $\alpha = -1$, $\alpha = 2$, $\alpha = 3$, $\alpha = 4$, and $\alpha = 5$, and does not pass in cases $\alpha = -3$, $\alpha = -2$, $\alpha = 1$. Its general solution, written in terms of quadrature, was found, as well as some exact solutions. Obtained exact solutions can be used as test functions in the analysis of the results of numerical modeling of processes in relaxing media described by equations of the Vakhnenko-Parkes type.

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